ERM-111-12
Key Rate Durations: Measures of Interest Rate Risks
Related Learning Objective
4i) Analyze funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Explain the concepts of immunization including modern refinements and practical limitations. Contrast the various risk measures and be able to apply these risk measures to various entities

Key Points of This Reading
1) Understand the advantages of key rate durations
2) Understand how to calculate the key rate durations
3) Understand the key rate durations for different instruments
4) Understand how to use the key rate durations for a portfolio
Effective Duration

It measures the price sensitivity of a bond to the change in interest rates (assume different points in the yield curve move in the same direction and magnitude) (parallel movement)

\[ \Delta P = -P \cdot D \cdot \Delta i \]

Change in Bond Price = \[ \Delta P = -P \cdot D \cdot \Delta i \]
Key Rate Duration

It measures the price sensitivity of a bond to each key rate (KR) change

Change in Bond Price (due to a change in KR) = $\Delta P_i = -P \cdot KRD_i \cdot \Delta KR_i$
The sum of key rate durations is equal to the effective duration

\[ D = \sum KRD_i \]

The total change in bond price

\[ \Delta P = -P \cdot \sum KRD_i \cdot \Delta KR_i \]

Positive Duration

\[ \Rightarrow \text{A decrease in interest rate will increase the bond price} \]

\[ \text{(many candidates get this wrong)} \]
Example

<table>
<thead>
<tr>
<th>Security</th>
<th>KRD(2-yr)</th>
<th>KRD(16-yr)</th>
<th>KRD(30-yr)</th>
<th>Effective Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>2yr zero</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>16yr zero</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>30yr zero</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Portfolio I: $50 MV of 2-year zero coupon bonds and $50 MV of 30-year zero coupon bonds

Portfolio II: $100 MV of 16-year zero coupon bonds

<table>
<thead>
<tr>
<th></th>
<th>KRD(2-yr)</th>
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<th>KRD(30-yr)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Portfolio I</td>
<td>1</td>
<td>0</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>Portfolio II</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>16</td>
</tr>
</tbody>
</table>
## Example

<table>
<thead>
<tr>
<th>Scenario</th>
<th>KRD(2-yr)</th>
<th>KRD(16-yr)</th>
<th>KRD(30-yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-10bps</td>
<td>-10bps</td>
<td>-10bps</td>
</tr>
<tr>
<td>2</td>
<td>+10bps</td>
<td>0bps</td>
<td>-10bps</td>
</tr>
<tr>
<td>3</td>
<td>-10bps</td>
<td>0bps</td>
<td>+10bps</td>
</tr>
</tbody>
</table>

Calculate the changes in portfolio values in these three scenarios.

**Portfolio 1 and Scenario 1**

\[
\Delta P = -P \cdot \sum KRD_i \cdot \Delta KR_i
\]

\[
= -50 \times (2 \times -0.1\% + 30 \times -0.1\%)
\]

\[
= 1.6
\]

**Portfolio 2 and Scenario 1**

\[
\Delta P = -P \cdot \sum KRD_i \cdot \Delta KR_i
\]

\[
= -100 \times 16 \times -0.1\%
\]

\[
= 1.6
\]

**Conclusion:** Same impact
## Example

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</table>

Calculate the changes in portfolio values in these three scenarios

\[
\Delta P = -P \cdot \sum KRD_i \cdot \Delta KR_i
\]

Portfolio 1 and Scenario 2

\[
\Delta P = -50 \times (2 \times 0.1\% + 30 \times -0.1\%)
\]

\[
= 1.4
\]

Portfolio 2 and Scenario 2

\[
\Delta P = -100 \times 16 \times 0\%
\]

\[
= 0
\]

Conclusion: No impact on portfolio 2
Example

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Calculate the changes in portfolio values in these three scenarios

\[
\Delta P = -P \cdot \sum KRD_i \cdot \Delta KRD_i
\]

Portfolio 1 and Scenario 3

\[
\Delta P = -50 \times (2 \times -0.1\% + 30 \times 0.1\%)
\]

\[
\Delta P = -1.4
\]

Portfolio 2 and Scenario 3

\[
\Delta P = -P \cdot \sum KRD_i \cdot \Delta KRD_i
\]

\[
\Delta P = -100 \times 16 \times 0\%
\]

\[
\Delta P = 0
\]

Conclusion: No impact on portfolio 2
Yield Curve Movements

- **Level Movement**

- **Steepness**

- **Curvature**
Key Rate Duration Profile

**Zero-Coupon Bond**
(ZCB)(20-Year)

![Graph for Zero-Coupon Bond](image)

**Coupon Bond**
(CB)(30-Year, 9%)

![Graph for Coupon Bond](image)

**Callable Corporate Bond**
(9% vs. 8% 30-Year)

![Graph for Callable Corporate Bond](image)

**Callable Bond With Sinking Fund**

![Graph for Callable Bond With Sinking Fund](image)
Key Rate Duration Profile

10-Year European Call/Put Option (on a 30-Year Bond)

Embedded Option in a Callable Bond (30-Year, 9%)

GNMA Pass-Through (30-Year Fixed Rate)

Interest-Only (IO) and Principal-Only (PO)
Use KRD to Identify Interest Rate Bets

Number of KRs = 11
Average KRD = 4.59 / 11 = 0.4173

<table>
<thead>
<tr>
<th>Term</th>
<th>0.25</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>KRD</td>
<td>0.02</td>
<td>0.19</td>
<td>0.35</td>
<td>0.56</td>
<td>0.69</td>
<td>0.74</td>
<td>0.65</td>
<td>0.61</td>
<td>0.34</td>
<td>0.28</td>
<td>0.16</td>
<td>4.59</td>
</tr>
</tbody>
</table>

Ratio(KRD) > 1.0 ➔ That means the index is more sensitive to that KRD

Ratio(0.25) = 0.02 / 0.4173 = 0.05
Ratio(1) = 0.19 / 0.4173 = 0.46
...
Ratio(30) = 0.16 / 0.4173 = 0.38
Use KRD to Create a Hedge Portfolio

Calculate the weight:

\[ W_i = \frac{KRD_i}{T_i} \]

The sum of the weights is equal to 1:

\[ 1 = \sum W_i \]

The value of the hedge portfolio:

\[ V \sum W_i = V \sum \frac{KRD_i}{T_i} \]

Invest \( VW_0 \) in cash

Invest \( \sum_{i=1}^{n} VW_i \) in zero-coupon bond
Use KRD to Create a Hedge Portfolio

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Suppose we have this KRD profile of a portfolio and want to create a hedge portfolio to hedge its interest rate exposure.

\[
V W_{0.25} = V \frac{KRD_{0.25}}{T_{0.25}} = 100 \times \frac{0.02}{0.25} = 8.0
\]
Invest $8.0 in three-month bills

\[
V W_1 = V \frac{KRD_1}{T_1} = 100 \times \frac{0.19}{1} = 19
\]
Invest $19.0 in 1-year ZCB

\[
V W_2 = V \frac{KRD_2}{T_2} = 100 \times \frac{0.35}{2} = 17.5
\]
Invest $17.5 in 2-year ZCB

<table>
<thead>
<tr>
<th>VW(1)</th>
<th>VW(2)</th>
<th>VW(3)</th>
<th>VW(4)</th>
<th>VW(5)</th>
<th>VW(6)</th>
<th>VW(7)</th>
<th>VW(8)</th>
<th>VW(9)</th>
<th>VW(10)</th>
<th>VW(11)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>8.0</td>
<td>19.0</td>
<td>17.5</td>
<td>18.7</td>
<td>13.8</td>
<td>10.6</td>
<td>6.5</td>
<td>4.1</td>
<td>1.7</td>
<td>1.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

\[
V \sum W_i = VW_0 + \sum_{i=1}^{n} VW_i
\]

\[
100 = VW_0 + 101.5
\]
Borrow $1.5 in cash

\[
VW_0 = -1.5
\]